

# Hierarchical Quantum Information Splitting with Six-Photon Cluster States

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**Abstract** We propose a scheme for hierarchical quantum information splitting with the recently realized six-photon cluster state (Lu et al. in *Nat. Phys.* 3:91, 2007), where a Boss distributes a quantum secret (quantum state) to five distant agents who are divided into two grades. Two agents are in the upper grade and three agents are in the lower grade. An agent of the upper grade only needs the collaboration of two of the other four agents for getting the secret, while an agent of the lower grade needs the collaboration of all the other four agents. In other words, the agents of two grades have different authorities to recover Boss's secret.

**Keywords** Quantum secret · Hierarchical splitting · Photonic qubit · Cluster states

The rapidly growing field of quantum information science is the fruit of the combination of information theory and quantum mechanics. Quantum information processing mainly involves the manipulation and transmission of information with the principle of quantum mechanics. The unique and useful properties of quantum mechanics is the inner reason for that why quantum information theory can implement many information processing tasks that classical information theory cannot achieve. In quantum information science, information is encoded in quantum states, and the information processing is in fact the manipulation and transfer of quantum states. Entanglement, the most intriguing property of quantum mechanics, is the center resource of quantum information science, and plays a powerful role in the transfer of quantum states. One well-known example is the quantum teleportation [1, 2] which utilizes the bipartite or multipartite entangled states to transport an unknown quantum state from one site to another one. However, not all entangled states can be used to implement perfect teleportation, and that whether or not an entangled state can implement teleportation is determined by its entanglement properties [3]. Thus teleportation can also reveal some properties of entangled states, especially multipartite entangled states [4, 5]. On the

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other hand, Multipartite entangled states may also implement the transfer of quantum states by other ways, instead of the way of one-to-one. With the Greenberger-Horne-Zeilinger (GHZ) states [6] Hillery et al. [7] firstly introduced the concept of quantum information splitting (QIS), where a quantum secret (quantum state) is distributed from a Boss to two or three distant agents such that any one of them can recover the secret with the assistance of the others. This idea can be directly generalized to the case of more than three agents [8]. QIS can be considered as a generalization of teleportation to more than one recipients, and also be referred to as open-destination teleportation or quantum-state sharing in literature [9–11].

QIS has extensive applications in quantum information science, such as creating joint checking accounts containing quantum money [12], secure distributed quantum computation [13], and so on. Since the end of last century, QIS has been attracting much attention [14–28], and a scheme has already been experimentally realized [9]. All of the aforementioned schemes are focused on the symmetric case where every agent has the same status, i.e., the same authority for getting the sender's secret. However, a more general QIS scheme should involve the asymmetry between the powers of the different participants. In Ref. [29], two of us introduced the concept of hierarchical QIS, in which the agents have different powers to recover the sender's secret, i.e., their authorities for getting the secret are hierarchized. Ref. [29] explicitly demonstrated a scheme for hierarchical QIS involving two grades with the upper grade containing one agent and the lower grade containing two agents. The scheme will be referred to as (1, 2)-hierarchy model of quantum information splitting. Ref. [29] also roughly discussed and analyzed the more general cases of more than three agents, and showed that the key point of designing a more general hierarchical QIS scheme is to find a suitable multipartite entangled state which can act as the quantum channel.

However, the construction of special structure of multipartite entangled states is not easy as pointed out in Ref. [29]. The experimental preparation of complicated multipartite entangled states is more challenging. Recently, a six-photon cluster state [30]

$$|C\rangle_6 = \frac{1}{2}(|H_1 H_2 H_3 H_4 H_5 H_6\rangle - |V_1 V_2 V_3 V_4 V_5 V_6\rangle + |V_1 V_2 V_3 H_4 H_5 H_6\rangle + |H_1 H_2 H_3 V_4 V_5 V_6\rangle) \quad (1)$$

has been successfully realized in experiment [31], where  $H$  and  $V$  denote, respectively, the horizontal and vertical polarizations, and the subscripts label the spatial modes of the photons. In this paper, we show that such a state can be used to implement (2, 3)-hierarchy model of QIS where two of five agents are in the upper grade and the other three are in the lower grade. Our scheme only involves a Bell-state measurement and several single-photon operations, and is experimentally achievable with current optical technology.

We consider that a Boss Alice share with her five agents (i.e., Bob<sup>1</sup>, Bob<sup>2</sup>, ..., Bob<sup>5</sup>) the six-photon cluster state of (1), where photon 1 is held by Alice and photon  $j$  ( $j = 2, 3, 4, 5, 6$ ) is held by Bob <sup>$j-1$</sup> . Alice has another photon 1' which is in the state

$$|\varphi\rangle_{1'} = \alpha|H_{1'}\rangle + \beta|V_{1'}\rangle \quad (2)$$

with  $|\alpha|^2 + |\beta|^2 = 1$ . The state of the whole system is

$$\begin{aligned} |\Phi\rangle_{whole} &= |\varphi\rangle_{1'} \otimes |C\rangle_6 \\ &= \frac{1}{\sqrt{2}}(\alpha|H_{1'} H_1\rangle|\mathcal{G}\rangle_{23456} + \alpha|H_{1'} V_1\rangle|\mathcal{G}'\rangle_{23456} \\ &\quad + \beta|V_{1'} H_1\rangle|\mathcal{G}\rangle_{23456} + \beta|V_{1'} V_1\rangle|\mathcal{G}'\rangle_{23456}), \end{aligned} \quad (3)$$

where

$$\begin{aligned}
 |\mathcal{G}\rangle_{23456} &= \frac{1}{\sqrt{2}}|H_2H_3\rangle(|H_4H_5H_6\rangle + |V_4V_5V_6\rangle), \\
 |\mathcal{G}'\rangle_{23456} &= \frac{1}{\sqrt{2}}|V_2V_3\rangle(|H_4H_5H_6\rangle - |V_4V_5V_6\rangle).
 \end{aligned}
 \tag{4}$$

The task is: Alice wants to distribute the state  $|\varphi\rangle$  to her five agents such that any one of them can recover the secret state with assistance of part or all of the others. To this end, Alice performs a joint measurement on her two photons 1' and 1 in the Bell basis  $\{|\Phi^\pm\rangle_{1'1}, |\Psi^\pm\rangle_{1'1}\}$ , and then informs them of the outcome. The four Bell states are given by

$$\begin{aligned}
 |\Phi^\pm\rangle_{1'1} &= \frac{1}{\sqrt{2}}(|H_{1'}H_1\rangle \pm |V_{1'}V_1\rangle), \\
 |\Psi^\pm\rangle_{1'1} &= \frac{1}{\sqrt{2}}(|H_{1'}V_1\rangle \pm |V_{1'}H_1\rangle).
 \end{aligned}
 \tag{5}$$

According to recent experiments [32–34], the Bell-state measurement for photonic qubits can be well realized. For Alice’s four possible outcomes,  $|\Phi^\pm\rangle_{1'1}$  or  $|\Psi^\pm\rangle_{1'1}$ , the photons held by Bobs collapse into the following corresponding entangled states:

$$\begin{aligned}
 |\phi^\pm\rangle_{23456} &= \alpha|\mathcal{G}\rangle_{23456} \pm \beta|\mathcal{G}'\rangle_{23456}, \\
 |\psi^\pm\rangle_{23456} &= \alpha|\mathcal{G}'\rangle_{23456} \pm \beta|\mathcal{G}\rangle_{23456}.
 \end{aligned}
 \tag{6}$$

The non-cloning theorem [35] allows only one photon to be in the state  $|\varphi\rangle$ , so that any one of the five agents (Bob<sup>1</sup>, Bob<sup>2</sup>, . . . , Bob<sup>5</sup>), but not all, can recover such a state.

First, we assume that they agree to let Bob<sup>1</sup> possess the secret. We rewrite  $|\phi^\pm\rangle_{23456}$  and  $|\psi^\pm\rangle_{23456}$  as

$$\begin{aligned}
 |\phi^\pm\rangle_{23456} &= \frac{1}{2}[(\alpha|H_2\rangle \pm \beta|V_2\rangle)(|+_3\rangle|H_4H_5H_6\rangle + |-_3\rangle|V_4V_5V_6\rangle) \\
 &\quad + (\alpha|H_2\rangle \mp \beta|V_2\rangle)(|+_3\rangle|V_4V_5V_6\rangle + |-_3\rangle|H_4H_5H_6\rangle), \\
 |\psi^\pm\rangle_{23456} &= \frac{1}{2}[(\alpha|V_2\rangle \pm \beta|H_2\rangle)(|+_3\rangle|H_4H_5H_6\rangle + |-_3\rangle|V_4V_5V_6\rangle) \\
 &\quad - (\alpha|V_2\rangle \mp \beta|H_2\rangle)(|+_3\rangle|V_4V_5V_6\rangle + |-_3\rangle|H_4H_5H_6\rangle),
 \end{aligned}
 \tag{7}$$

where  $|\pm\rangle = (|H\rangle \pm |V\rangle)/\sqrt{2}$ . In order to assist Bob<sup>1</sup> in recovering Alice’s secret state, the other Bobs need to measure their photons in suitable bases and broadcast the outcomes. Obviously, if Bob<sup>3</sup>, Bob<sup>4</sup>, and Bob<sup>5</sup> measure their photons in the basis  $\{|H\rangle, |V\rangle\}$ , their outcomes are always correlated, i.e., anyone’s outcome can deduce the others’. This implies that only one of them, to be referred to as “Bob\*”, is needed to inform Bob<sup>1</sup> of his single-photon measurement outcome. Suppose that Bob<sup>2</sup> measures his photon 3 in the basis  $\{|+\rangle, |-\rangle\}$ . Then Bob<sup>1</sup> can reconstruct the state  $|\varphi\rangle$  on photon 2 by appropriate local operations (unitary transformations) based on the measurement outcomes of Bob<sup>2</sup> and Bob\*. In other words, the two agents Bob<sup>2</sup> and Bob\* are sufficient to assist Bob<sup>1</sup> in recovering Alice’s secret state  $|\varphi\rangle$ . The corresponding local operations that Bob<sup>1</sup> should perform on photon 2 are listed in Table 1, where  $I$  is the identity operator, and  $\sigma^x = |H\rangle\langle V| + |V\rangle\langle H|$  and  $\sigma^z = |H\rangle\langle H| - |V\rangle\langle V|$  are the usual Pauli operators. The Pauli operators for photonic

**Table 1** The corresponding local operations that Bob<sup>1</sup> should perform for recovering the state  $|\varphi\rangle$ , according to the measurement outcomes of Alice, Bob<sup>2</sup>, and Bob\*

Alice's outcomes	Bob <sup>2</sup> 's outcomes	Bob*'s outcomes	Bob <sup>1</sup> 's operations
$ \Phi^+\rangle$	$ +\rangle( -\rangle)$	$ H\rangle( V\rangle)$	$I$
$ \Phi^+\rangle$	$ +\rangle( -\rangle)$	$ V\rangle( H\rangle)$	$\sigma^z$
$ \Phi^-\rangle$	$ +\rangle( -\rangle)$	$ H\rangle( V\rangle)$	$\sigma^z$
$ \Phi^-\rangle$	$ +\rangle( -\rangle)$	$ V\rangle( H\rangle)$	$I$
$ \Psi^+\rangle$	$ +\rangle( -\rangle)$	$ H\rangle( V\rangle)$	$\sigma^x$
$ \Psi^+\rangle$	$ +\rangle( -\rangle)$	$ V\rangle( H\rangle)$	$\sigma^x\sigma^z$
$ \Psi^-\rangle$	$ +\rangle( -\rangle)$	$ H\rangle( V\rangle)$	$\sigma^x\sigma^z$
$ \Psi^-\rangle$	$ +\rangle( -\rangle)$	$ V\rangle( H\rangle)$	$\sigma^x$

qubits can be easily realized with linear optical elements (e.g., half-wave plates, quarter-wave plates, phase shifters, etc.). The above results are also applicable to the case where Bob<sup>2</sup> is deputed to possess Alice's secret because  $|\mathcal{C}\rangle_6$  is unchanged under the permutation of photons 2 and 3, which indicates that Bob<sup>1</sup> and Bob<sup>2</sup> have the same status in the QIS protocol.

Now, we consider the case that they agree to let Bob<sup>5</sup> possess Alice's secret, i.e., recover the state  $|\varphi\rangle$ . Then the states  $|\phi^\pm\rangle_{23456}$  and  $|\psi^\pm\rangle_{23456}$  can be rewritten as

$$\begin{aligned}
 |\phi^\pm\rangle_{23456} &= \frac{1}{4}[(|+\rangle_2|+\rangle_3|+\rangle_4|+\rangle_5) + |+\rangle_2|+\rangle_3|-\rangle_4|-\rangle_5) + |-\rangle_2|-\rangle_3|+\rangle_4|+\rangle_5) \\
 &\quad + |-\rangle_2|-\rangle_3|-\rangle_4|-\rangle_5)(\alpha|+_6\rangle \pm \beta|-_6\rangle) \\
 &\quad + (|+\rangle_2|-\rangle_3|+\rangle_4|+\rangle_5) + |-\rangle_2|+\rangle_3|+\rangle_4|+\rangle_5) + |+\rangle_2|-\rangle_3|-\rangle_4|-\rangle_5) \\
 &\quad + |-\rangle_2|+\rangle_3|-\rangle_4|-\rangle_5)(\alpha|+_6\rangle \mp \beta|-_6\rangle) \\
 &\quad + (|+\rangle_2|+\rangle_3|+\rangle_4|-\rangle_5) + |+\rangle_2|+\rangle_3|-\rangle_4|+\rangle_5) + |-\rangle_2|-\rangle_3|+\rangle_4|-\rangle_5) \\
 &\quad + |-\rangle_2|-\rangle_3|-\rangle_4|+\rangle_5)(\alpha|-_6\rangle \pm \beta|+_6\rangle) \\
 &\quad + (|+\rangle_2|-\rangle_3|+\rangle_4|-\rangle_5) + |-\rangle_2|+\rangle_3|+\rangle_4|-\rangle_5) + |+\rangle_2|-\rangle_3|-\rangle_4|+\rangle_5) \\
 &\quad + |-\rangle_2|+\rangle_3|-\rangle_4|+\rangle_5)(\alpha|-_6\rangle \mp \beta|+_6\rangle)], \tag{8}
 \end{aligned}$$

$$\begin{aligned}
 |\psi^\pm\rangle_{23456} &= \frac{1}{4}[(|+\rangle_2|+\rangle_3|+\rangle_4|-\rangle_5) + |+\rangle_2|+\rangle_3|-\rangle_4|+\rangle_5) + |-\rangle_2|-\rangle_3|+\rangle_4|-\rangle_5) \\
 &\quad + |-\rangle_2|-\rangle_3|-\rangle_4|+\rangle_5)(\alpha|+_6\rangle \pm \beta|-_6\rangle) \\
 &\quad - (|+\rangle_2|-\rangle_3|+\rangle_4|-\rangle_5) + |+\rangle_2|-\rangle_3|-\rangle_4|+\rangle_5) + |-\rangle_2|+\rangle_3|+\rangle_4|-\rangle_5) \\
 &\quad + |-\rangle_2|+\rangle_3|-\rangle_4|+\rangle_5)(\alpha|+_6\rangle \mp \beta|-_6\rangle) \\
 &\quad + (|+\rangle_2|+\rangle_3|+\rangle_4|+\rangle_5) + |+\rangle_2|+\rangle_3|-\rangle_4|-\rangle_5) + |-\rangle_2|-\rangle_3|+\rangle_4|+\rangle_5) \\
 &\quad + |-\rangle_2|-\rangle_3|-\rangle_4|-\rangle_5)(\alpha|-_6\rangle \pm \beta|+_6\rangle) \\
 &\quad - (|-\rangle_2|+\rangle_3|+\rangle_4|+\rangle_5) + |+\rangle_2|-\rangle_3|+\rangle_4|+\rangle_5) + |+\rangle_2|-\rangle_3|-\rangle_4|-\rangle_5) \\
 &\quad + |-\rangle_2|+\rangle_3|-\rangle_4|-\rangle_5)(\alpha|-_6\rangle \mp \beta|+_6\rangle)].
 \end{aligned}$$

**Table 2** The corresponding local operations that Bob<sup>5</sup> should perform for recovering the state  $|\varphi\rangle$ , according to the measurement outcomes of Boss Alice and the other four agents (Bob<sup>1</sup>, Bob<sup>2</sup>, Bob<sup>3</sup>, Bob<sup>4</sup>)

The outcomes of Alice	The outcomes of Bob <sup>1,2,3,4</sup>	The operations of Bob <sup>5</sup>
$ \Phi^+\rangle ( \Phi^-\rangle)$	$ ++++\rangle$ or $ + + - -\rangle$ or $  - - + +\rangle$ or $  - - - -\rangle$	$\mathcal{H} (\sigma^z \mathcal{H})$
$ \Phi^+\rangle ( \Phi^-\rangle)$	$ + - + +\rangle$ or $  - + + +\rangle$ or $  + - - -\rangle$ or $  - + - -\rangle$	$\sigma^z \mathcal{H} (\mathcal{H})$
$ \Phi^+\rangle ( \Phi^-\rangle)$	$  + + + -\rangle$ or $  + + - +\rangle$ or $  - - + -\rangle$ or $  - - - +\rangle$	$\sigma^x \mathcal{H} (\sigma^x \sigma^z \mathcal{H})$
$ \Phi^+\rangle ( \Phi^-\rangle)$	$  + - + -\rangle$ or $  - + + -\rangle$ or $  + - - +\rangle$ or $  - + - +\rangle$	$\sigma^x \sigma^z \mathcal{H} (\sigma^x \mathcal{H})$
$ \Psi^+\rangle ( \Psi^-\rangle)$	$  + + + -\rangle$ or $  + + - +\rangle$ or $  - - + -\rangle$ or $  - - - +\rangle$	$\mathcal{H} (\sigma^z \mathcal{H})$
$ \Psi^+\rangle ( \Psi^-\rangle)$	$  + - + -\rangle$ or $  + - - +\rangle$ or $  - + + -\rangle$ or $  - + - +\rangle$	$\sigma^z \mathcal{H} (\mathcal{H})$
$ \Psi^+\rangle ( \Psi^-\rangle)$	$  + + + +\rangle$ or $  + + - -\rangle$ or $  - - + +\rangle$ or $  - - - -\rangle$	$\sigma^x \mathcal{H} (\sigma^x \sigma^z \mathcal{H})$
$ \Psi^+\rangle ( \Psi^-\rangle)$	$  - + + +\rangle$ or $  + - + +\rangle$ or $  + - - -\rangle$ or $  - + - -\rangle$	$\sigma^x \sigma^z \mathcal{H} (\sigma^x \mathcal{H})$

It can be seen that Bob<sup>5</sup> can reconstruct the state  $|\varphi\rangle$  if and only if all the other Bobs measure their photons in the basis  $\{|+\rangle, |-\rangle\}$  and broadcast their outcomes. In other words, Bob<sup>5</sup> needs the assistance of all of the other four agents (Bob<sup>1</sup>, Bob<sup>2</sup>, Bob<sup>3</sup>, Bob<sup>4</sup>) for recovering the secret. The corresponding local operations that Bob<sup>5</sup> should make, according to Alice’s Bell-state measurement outcomes and the other Bobs’ single-qubit measurement outcomes, are listed in Table 2, where  $\mathcal{H} = |H\rangle\langle H| + |H\rangle\langle V| + |V\rangle\langle H| - |V\rangle\langle V|$  is the Hadamard transformation (which can be easily realized by a quarter-wave plate). These results are also applicable to the case where Bob<sup>3</sup> or Bob<sup>4</sup> is deputed to recover Alice’s secret because  $|C\rangle_6$  is unchanged under the permutation of photons 4, 5, and 6, which indicates that Bob<sup>3</sup>, Bob<sup>4</sup>, and Bob<sup>5</sup> have the same status in the QIS protocol.

In a word, for recovering the secret state  $|\varphi\rangle$ , Bob<sup>1</sup> (Bob<sup>2</sup>) only needs the assistance of Bob<sup>2</sup> (Bob<sup>1</sup>) and any one of the other three Bobs (i.e., Bob<sup>3,4,5</sup>), while one of Bob<sup>3,4,5</sup> needs the help of all of the other Bobs. Thus, their authorities for getting the secret are hierarchized, and Bob<sup>1,2</sup> are in a higher grade relative to Bob<sup>3,4,5</sup>. This result may be understood from the following picture. After Alice’s Bell-state measurement with outcomes  $|\Phi^\pm\rangle_{1'1}$  or  $|\Psi^\pm\rangle_{1'1}$ , Bobs’ single-photon state-density matrices are, respectively,

$$\begin{aligned}
 \rho_{2(3)}^{|\Phi^\pm\rangle} &= |\alpha|^2 |H\rangle_{2(3)} \langle H| + |\beta|^2 |V\rangle_{2(3)} \langle V|, \\
 \rho_{2(3)}^{|\Psi^\pm\rangle} &= |\beta|^2 |H\rangle_{2(3)} \langle H| + |\alpha|^2 |V\rangle_{2(3)} \langle V|, \\
 \rho_{4(5,6)} &= \frac{1}{2} (|H\rangle_{4(5,6)} \langle H| + |V\rangle_{4(5,6)} \langle V|),
 \end{aligned}
 \tag{9}$$

where the superscripts  $|\Phi^\pm\rangle$  and  $|\Psi^\pm\rangle$  denote Alice’s measurement outcomes. It can be seen that Bob<sup>3</sup>, Bob<sup>4</sup> or Bob<sup>5</sup> knows nothing about the information of Alice’s secret state  $|\varphi\rangle$  without the collaboration of the other Bobs; Bob<sup>1</sup> or Bob<sup>2</sup>, however, has the amplitude information of  $|\varphi\rangle$  as long as receiving Alice’s Bell-state measurement outcome. This case implies that Alice’s quantum secret is distributed to Bob<sup>1,2</sup> and Bob<sup>3,4,5</sup> asymmetrically. Naturally, the more information is known, the less collaborations are needed.

The above QIS protocol can also be modified to implement controlled teleportation [2] if we assume that Bob<sup>1</sup> and Bob<sup>2</sup> denote the same agent Bob\* and he is chosen as the receiver in advance. The scheme of Ref. [2] is a (2, 2)-threshold controlling scheme where the achievement of teleportation is conditioned on the collaboration of both of the two supervisors, while the present one is a (1, 3)-threshold controlling scheme where one of the three

supervisors (Bob<sup>3</sup>, Bob<sup>4</sup>, and Bob<sup>5</sup>) can help Bob\* successfully recover the teleported state. Very recently, several other types of  $(k, m)$ -threshold-controlled teleportation protocols have also been proposed [36–38].

In conclusion, we have proposed a scheme for (2, 3)-hierarchy model of QIS with the experimentally realized six-photon cluster state [31], where the five agents (Bob<sup>1,2,3,4,5</sup>) are divided into two grades. Two agents (Bob<sup>1,2</sup>) are in the upper grade and three agents (Bob<sup>3,4,5</sup>) are in the lower grade. The agents of different grades have different authorities for getting Boss's (Alice's) quantum secret. Bob<sup>1</sup> (Bob<sup>2</sup>) only needs the assistance of Bob<sup>2</sup> (Bob<sup>1</sup>) and any one of the other three Bobs for recovering the secret; while one of Bob<sup>3,4,5</sup> needs the help of all of the other Bobs. That is, Bob<sup>1</sup> and Bob<sup>2</sup> have a larger authority than Bob<sup>3,4,5</sup> to possess the final secret. The presented hierarchical QIS scheme can be experimentally realized with the techniques of recent experiments [31–34]. Our scheme can also be modified to implement (1, 3)-threshold controlled teleportation.

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